## Aerodynamics <br> Formula Overview

## Equation of state

$$
\begin{equation*}
p=\rho R T \tag{1}
\end{equation*}
$$

Relationship between gravity with geometric height

$$
\begin{equation*}
g=g_{0}\left(\frac{r}{r+h_{G}}\right)^{2} \tag{2}
\end{equation*}
$$

## Hydrostatic equation

Geometric altitude:

$$
\begin{equation*}
d p=-\rho g d h_{G} \tag{3}
\end{equation*}
$$

Geopotential altitude:

$$
\begin{equation*}
d p=-\rho g_{0} d h \tag{4}
\end{equation*}
$$

Relationship geopotential and geometric altitudes

$$
\begin{equation*}
d h=\frac{r^{2}}{\left(r+h_{G}\right)^{2}} d h_{G} \Rightarrow h=\frac{r}{r+h_{G}} h_{G} \tag{5}
\end{equation*}
$$

## Standard atmosphere

Isothermal layer

$$
\begin{equation*}
\frac{p}{p_{1}}=\frac{\rho}{\rho_{1}}=e^{-\frac{g_{0}\left(h-h_{1}\right)}{R T}} \tag{6}
\end{equation*}
$$

Gradient layer

$$
\begin{gather*}
\frac{p}{p_{1}}=\left(\frac{T}{T_{1}}\right)^{-\frac{g_{0}}{a R}}  \tag{7}\\
\frac{\rho}{\rho_{1}}=\left(\frac{T}{T_{1}}\right)^{-\left(\frac{g_{0}}{a R}+1\right)}  \tag{8}\\
T=T_{1}+a\left(h-h_{1}\right) \tag{9}
\end{gather*}
$$

Euler equation

$$
\begin{equation*}
d p=-\rho V d V \tag{10}
\end{equation*}
$$

Steady frictionless incompressible flow Simplified continuity equation:

$$
\begin{equation*}
A_{1} V_{1}=A_{2} V_{2} \tag{11}
\end{equation*}
$$

Bernoulli equation:

$$
\begin{equation*}
p_{1}+\frac{1}{2} \rho V_{1}^{2}=p_{2}+\frac{1}{2} \rho V_{2}^{2} \tag{12}
\end{equation*}
$$

## Curved flow

$$
\begin{equation*}
\frac{d p}{d r}=\rho \frac{V^{2}}{r} \tag{13}
\end{equation*}
$$

## Thermodynamics

Enthalpy:

$$
\begin{equation*}
h=e+p v=e+R T \tag{14}
\end{equation*}
$$

First law of thermodynamics:

$$
\begin{equation*}
\delta q+\delta w=d e \tag{15}
\end{equation*}
$$

At constant pressure:

$$
\begin{equation*}
\delta q=d e+p d v \tag{16}
\end{equation*}
$$

At constant volume:

$$
\begin{equation*}
\delta q=d h-v d p \tag{17}
\end{equation*}
$$

Specific heat at constant volume:

$$
\begin{equation*}
c_{v}=\frac{\delta q}{d T} \tag{18}
\end{equation*}
$$

Specific heat at constant pressure:

$$
\begin{equation*}
c_{p}=\frac{\delta q}{d T} \tag{19}
\end{equation*}
$$

$$
\begin{gather*}
d e=c_{v} d T \Rightarrow e=c_{v} T  \tag{20}\\
d h=c_{p} d T \Rightarrow h=c_{p} T  \tag{21}\\
R=c_{p}-c_{v}  \tag{22}\\
\gamma=\frac{c_{p}}{c_{v}} \tag{23}
\end{gather*}
$$

Mach number

$$
\begin{gather*}
M=\frac{V}{a}  \tag{24}\\
a=\sqrt{\gamma R T} \tag{25}
\end{gather*}
$$

## Isentropic flow

Continuity equation:

$$
\begin{gather*}
\rho_{1} A_{1} V_{1}=\rho_{2} A_{2} V_{2}  \tag{26}\\
\frac{p_{2}}{p_{1}}=\left(\frac{\rho_{2}}{\rho_{1}}\right)^{\gamma}=\left(\frac{T_{2}}{T_{1}}\right)^{\frac{\gamma}{\gamma-1}}  \tag{27}\\
\frac{T_{0}}{T_{1}}=1+\frac{\gamma-1}{2} M_{1}^{2}  \tag{28}\\
\frac{p_{0}}{p_{1}}=\left(1+\frac{\gamma-1}{2} M_{1}^{2}\right)^{\frac{\gamma}{\gamma-1}}  \tag{29}\\
\frac{\rho_{0}}{\rho_{1}}=\left(1+\frac{\gamma-1}{2} M_{1}^{2}\right)^{\frac{1}{\gamma-1}}  \tag{30}\\
c_{p} T_{1}+\frac{1}{2} V_{1}^{2}=c_{p} T_{2}+\frac{1}{2} V_{2}^{2} \tag{31}
\end{gather*}
$$

Measurement of airspeed
Incompresible flow ( $M<0.3$ ):

$$
\begin{align*}
V_{\text {true }} & =\sqrt{\frac{2\left(p_{0}-p\right)}{\rho}}  \tag{32}\\
V_{e} & =\sqrt{\frac{2\left(p_{0}-p\right)}{\rho_{s}}} \tag{33}
\end{align*}
$$

Subsonic compressible flow $(0.3<M<1)$ :

$$
\begin{align*}
V_{1}^{2} & =\frac{2 a_{1}^{2}}{\gamma-1}\left[\left(\frac{p_{0}-p_{1}}{p_{1}}+1\right)^{\frac{\gamma-1}{\gamma}}-1\right]  \tag{34}\\
V_{c a l}^{2} & =\frac{2 a_{1}^{2}}{\gamma-1}\left[\left(\frac{p_{0}-p_{1}}{p_{s}}+1\right)^{\frac{\gamma-1}{\gamma}}-1\right] \tag{35}
\end{align*}
$$

Relationship true airspeed and equivalent airspeed:

$$
\begin{equation*}
V_{e}=V \sqrt{\frac{\rho}{\rho_{s}}} \tag{36}
\end{equation*}
$$

## Area velocity relation

$$
\begin{equation*}
\frac{d A}{A}=\left(M^{2}-1\right) \frac{d V}{V} \tag{37}
\end{equation*}
$$

Conclusions:
$M<1$ : For the velocity to increase, the area must decrease
$M>1$ : For the velocity to increase, the area must increase
$M=1$ : The velocity will be sonic always at the throat

## Reynold's number

$$
\begin{equation*}
R e_{x}=\frac{\rho_{\infty} V_{\infty} x}{\mu_{\infty}} \tag{38}
\end{equation*}
$$

## Viscous flow

$$
\begin{align*}
\tau_{w} & =\mu\left(\frac{d V}{d y}\right)_{y=0}  \tag{39}\\
c_{f_{x}} & =\frac{\tau_{w}}{\frac{1}{2} \rho_{\infty} V_{\infty}^{2}}=\frac{\tau_{w}}{q_{\infty}} \tag{40}
\end{align*}
$$

Laminar boundary layer:

$$
\begin{gather*}
\delta=\frac{5.2 x}{\sqrt{R e_{x}}}  \tag{41}\\
c_{f_{x}}=\frac{0.664}{\sqrt{R e_{L}}}  \tag{42}\\
C_{f}=\frac{D_{f}}{q_{\infty} S}=\frac{1.328}{\sqrt{R e_{L}}} \tag{43}
\end{gather*}
$$

Turbulent boundary layer

$$
\begin{equation*}
\delta=\frac{0.37 x}{R e_{x}^{0.2}} \tag{44}
\end{equation*}
$$

$$
\begin{gather*}
c_{f_{x}}=\frac{0.0592}{R e_{L}^{0.2}}  \tag{45}\\
C_{f}=\frac{D_{f}}{q_{\infty} S}=\frac{0.074}{R e_{L}^{0.2}} \tag{46}
\end{gather*}
$$

Drag due to viscous effects:

$$
\begin{equation*}
D_{\text {profile }}=D_{\text {friction }}+D_{\text {pressure }} \tag{47}
\end{equation*}
$$

## Airfoil nomenclature

Relation lift and drag force with normal and axial force:

$$
\begin{align*}
& L=N \cos \alpha-A \sin \alpha  \tag{48}\\
& D=N \sin \alpha+A \cos \alpha \tag{49}
\end{align*}
$$

$\mathrm{AC}, \mathrm{CP}$ and the pitching moment

$$
\begin{gather*}
C_{m, C P}=0  \tag{50}\\
\frac{d C_{m, A C}}{d \alpha}=0  \tag{51}\\
C_{m Q_{2}}=C_{m Q_{1}}+C_{n}\left(\frac{x_{Q_{2}}-x_{Q_{1}}}{c}\right) \tag{52}
\end{gather*}
$$

Lift, drag and moment for 2-dimensional wings

$$
\begin{gather*}
L=q_{\infty} S c_{l}  \tag{53}\\
D=q_{\infty} S c_{d}  \tag{54}\\
M=q_{\infty} S c c_{m} \tag{55}
\end{gather*}
$$

Pressure coefficient:

$$
\begin{equation*}
C_{p}=\frac{p-p_{\infty}}{q_{\infty}}=\frac{p-p_{\infty}}{\frac{1}{2} \rho_{\infty} V_{\infty}^{2}} \tag{56}
\end{equation*}
$$

Prandt-glauert rule:

$$
\begin{gather*}
C_{p}=\frac{C_{p, 0}}{\sqrt{1-M_{\infty}^{2}}}  \tag{57}\\
c_{n}=\frac{1}{c} \int_{0}^{c}\left(C_{p, l}-C_{p, u}\right) d x \tag{58}
\end{gather*}
$$

Induced drag for 3-dimensional wings:

$$
\begin{gather*}
D_{i}=L \alpha_{i}=L \frac{C_{L}}{\pi A}  \tag{59}\\
C_{D, i}=\frac{C_{L}^{2}}{\pi e A} \tag{60}
\end{gather*}
$$

( $e=1$ for elliptical shapes)

$$
\begin{gather*}
A=\frac{b}{c}=\frac{b^{2}}{S}  \tag{61}\\
\frac{d C_{L}}{d \alpha}=a=\frac{a_{0}}{1+\frac{57.3 a_{0}}{\pi e_{1} A}}  \tag{62}\\
D_{\text {total }}=D_{\text {profile }}+D_{\text {induced }} \tag{63}
\end{gather*}
$$

## Mach waves

$$
\begin{gather*}
\mu=\arcsin \frac{1}{M}  \tag{64}\\
M_{c r, s w e p t}=\frac{M_{c r, a i r f o i l}}{\cos \Omega} \tag{65}
\end{gather*}
$$

